Venture Capital Financing : A Theoretical Model

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This paper investigates venture capital financing in an environment where a manager faces an imperfect capital market and a venture capitalist faces a moral hazard and uncertainty. Active monitoring, staged funding and syndication can alleviate moral hazard and reduce risk. At first stage, we compare the social welfare in case of up-front funding with monitoring, and in case of staged funding without monitoring. We find that up-front funding can provide greater benefits than in case of staged funding. Moreover, under given conditions, we find that staged funding acts as a successful device in controlling information asymmetries. At a second stage, we find that syndicated investments used with active monitoring gives the higher profit to the start-up due to the VCs free-riding problem in monitoring.

INTRODUCTION

Venture capital is often the source of financing in the essential stages of the early development of many firms. The fundamental need for funding in start-up firms comes from the entrepreneur's wealth constraints. One important characteristic of many start-ups is the high risk due to the great uncertainty about returns, the lack of substantial tangible assets and the lack of a track record in operations. Many start-ups may face many years of negative earnings before they start to see profits. Given this situation, banks and other intermediaries are averse to or even prohibited from lending money to such firms. Moreover, these financial intermediaries usually lack expertise in investing in young and high-risk companies. Consequently, these start-ups often seek venture capitalists to be involved in their activities by offering revenue sharing in the form of equity joint ventures in order to obtain necessary funding and to benefit from the venture capitalists 'experience in management and finance (Wang and Zhou, 2004).

Venture capital firms are financial intermediaries focused on providing capital to small, fast-growth start-up companies that are typically high risk and not amenable to more traditional financing alternatives. In comparison with traditional sources of funds, venture capital firms have some particular characteristics. According to Gompers (1995), venture capital's investments entail higher intensities of uncertainty, asymmetric information and in general higher intangible

assets and development opportunities. Sapienza and Gupta (1994) assert that venture capitalists obtain an equity position in the corporation and play an active role in the corporate governance. In other words, they sit in the board of directors and regularly monitor performance (Shalman, 1990). Key characteristics in venture capital financing are staging the commitment of capital and preserving the option to abandon the project. Instead of providing all the necessary capital upfront, venture capitalists invest in stages to keep the project under control. Staged investment allows venture capitalists to monitor the firm before they make refinancing decisions. The information about the viability of a project acquired through such monitoring helps venture capitalists to avoid throwing money at bad projects. It reduces losses from inefficient continuation and creates an exit option for venture capitalists.

In this paper, we investigate venture capital financing in an environment where an entrepreneur faces an imperfect capital market and an investor faces moral hazard and uncertainty. We consider an up-front funding, and then we consider a stage funding and syndication, to compare the effects of these financing strategies on the manager's behavior.
Staged financing can play two roles in this model: to control risk and to alleviate moral hazard. Staged financing has been widely used in venture capital, especially in the United States.

Can staged financing be used by venture capitalists to reduce problems of asymmetry information? To answer this question, we present a comprehensive, two stages model of the venture capital contracting process, incorporating moral hazard and asymmetric information problems. The structure of the model involving managerial effort, staged investment reproduces what we know of venture capital financing. Using parametric functions, we discover a few interesting properties of staged financing. In particular, we show that when used together with a sharing contract, staged financing acts as an effective complementary mechanism to contracting in controlling agency problems. Moreover, venture capitalists often band together in groups to invest in companies. Conventional wisdom says they engage in this practice, called syndication, to better screen potential investments. More eyes the thinking goes yield better investments (Amit, 2003).

From the lead venture capitalist's point of view, the benefit of seeking syndication is that the value of the project rises if other venture capitalists become involved. Syndication, for example, lets them to share risk and diversify their portfolios, Amit found some evidence of that. Banding together might also give them a stronger bargaining position vis-à-vis the companies in which they invest. Formally, we consider a financially constrained manager and a venture capitalist who is interested in investing in the manager's project. The project is risky and the manager's effort is not verifiable. We assume that the manager faces an imperfect capital market and the venture capital is the only potential investor who accepts to finance the project. The venture capital offers a sharing contract and finances the project strategically in stages in one hand, and syndicated the investment on the other hand. Using parametric functions, we are able to derive some interesting properties of venture capital financing. Our results show that, in addition to contracting, staged financing is an effective mechanism for venture capitalists to reduce problems of asymmetry information and to control risks resulting from the manager's behavior. By taking into account the active monitoring role of the venture capitalist we show that up-front funding provides a higher profit than in case of staged funding, or syndication.

LITERATURE REVIEW

An important source of financing for the entrepreneurial sector is the venture capital industry. Indeed, venture capital activity is usually defined as the provision of equity financing to young firms. In addition, they often provide managerial advice to their investees. The venture capital industry is characterized by great uncertainty about returns and information asymmetries between principals and agents. The structure of this section applies some aspects of the theory of asymmetric information to a financial contracting setting in which an entrepreneur may obtain funding from a venture capitalist. Moral hazard problems are particularly important in situations where one party acts as an agent for another party: the principal. In these situations, the principal can't perfectly observe the effort (or other actions) of the agent. Jensen and Meckling [1976] argued that agency analysis was the key to understanding the modern firm. Other classic papers on the agency problem include Holmstrom [1979] and Grossman and Hart [1983].
The role of asymmetry of information in financial contracting in venture capital is commonly recognizable.

Shalman [1990] postulated that contracting practices in the venture capital industry reveal uncertainty about payoffs and information asymmetries between venture capitalists and managers. He precisely argued that the lack of operational history intensifies the adverse selection dilemma. During the screening stage, venture capitalist reviews business plans of start-ups and proposes to the manager contracts that reduce agency costs. Shalman [1990] considers venture capital as control mechanisms used to alleviate agency costs.
Gompers [1995] asserts that "three control mechanisms are common to nearly all venture capital financing:

- The use of convertible securities;
- The syndication of investment;
- The staging of capital infusions.

Some researchers suggested solutions engineered by the venture capital industry to overcome problems arising from the asymmetry of information include the use of convertible preferred stock (Bary, 1994), Trester (1998) control rights (Hellmann, 1998) which may include representation on the boards of start-ups (Lerner, 1995) and syndication (Lerner, 1994), Brander, Amit and Antweiler (1995). The syndication may be a mechanism through which venture capitalists determine informational uncertainties about potential investments. Admati and Pfleiderer (1994) develop a rationale for syndication in later venture rounds that is based on informational asymmetries between the initial venture investor and other potential investors. A venture capitalist who is involved in the firm's operations may utilize this informational advantage, overstating the appropriate price for the securities in the next financing round. The only way to avoid this opportunistic behavior is when the lead venture capitalist maintains a constant share of the firm's equity. Syndication may also be a mechanism through which venture capitalists exploit informational asymmetries and collude to overstate their performance to potential investors.

Venture capitalists prefer syndicating most deals for a simple reason; it means they have a chance to check out their own thinking against other knowledgeable. If two or three other funds whose thinking you respect agree to go along, that is a double check to your own thinking. Most financing involves a syndicate of two or more venture groups, providing more capital availability

for current and follow-on cash needs. Syndication also spreads the risk and brings together more expertise and support. These benefits pertain only to start-up financing requiring the venture capitalist's first investment decision. There are different strategies and motivations for syndication in follow-on financing. Lead venture capitalists who become involved in the firm's operations can solve this information problem. Other less well-informed investors will invest if this lead one does. Venture capitalists, however, may exploit their informational advantage and overstate the proper price for the securities in the firm's financing. Staged investment, which creates the option to abandon a venture, is an important mean for venture capitalists to minimize the present value of agency costs. The active involvement of venture capitalists in the operations of their invested companies may also mitigate the moral hazard problem.

Staging capital allows the venture capitalist to gather information and monitor the progress of the firms with maintaining the option to periodically abandon projects. The role of staged funding is similar to that of debt in highly leveraged transactions, keeping the owner/manager on a "tight leash" and reducing potential losses from bad decisions. Total venture financing and the number of financing rounds should also be higher for successful projects than for failures if venture capitalist uses information in investment decisions. In other words, venture capitalists monitor firm's progress and discontinue funding the project if they learn negative information about future prospects. Amit, Glosten and Muller (1990a) related the venture capital financing decision to the entrepreneur's skill level and predicted which entrepreneurs would decide to enter into an agreement with venture capitalists. They also considered moral hazard problems, but treated adverse selection only in the situation where the entrepreneur's type becomes common knowledge between hiding and contracting. Hirao (1993) assumed that the agent's (the manager's) unobservable actions affect the learning process. She found that because of the interaction between learning and moral hazard, a long-term contract is not equivalent to a series of short term contracts. Bergemann and Hege (1998) analyzed a similar situation in which there is a link between moral hazard and gradual learning about project quality. They assumed that an entrepreneur can't affect the up-side potential of her venture; she can only prevent intrinsically good projects from failing. Withholding effort (moral hazard) not only endangers the success of the venture but also causes the entrepreneur's and the venture capitalist (initially symmetric) learning about project quality to diverge. Bergemann and Hege (1998) found that the optimal contract in this situation is a time varying share contract. Amit, Glosten and Muller (1990b) considered the role of different mechanisms for matching entrepreneurs and venture capitalists in mitigating adverse selection problems. In their work, entrepreneurs are assumed to have private information about their types and venture capitalists can participate in the management of invested firms (at some cost) and thus contribute directly to the venture's success.

Managers may "stop around" or venture capitalists may actively seek out attractive investment opportunities. With a three-stage game, the authors examined possible pooling and separating equilibrium. To characterize the contract that allows optimal continuation decisions with a staged finance, Admati and Pfleiderer (1994) found that venture capitalists should prefer a fixed fraction contract. This contract stipulates that the venture capitalist owns a certain fraction of the final payoffs, and finances the same fraction of any future investment (if the continuation of the project is desirable). This result explains why later stages are not fully financed by the lead venture capitalist. It also attributes a positive role to the venture capitalist as a financial intermediary between the manager and outside investors.

On the same direction, Hellmann (1994) built a multi-stage model involving staged investment. He explained certain institutional features that distinguish venture capital from more

traditional methods of finance. For example, he explained that only when the venture capitalist has a concentrated stake in invested companies will there be a sufficiently high incentive for active monitoring. The monitoring goes beyond what a traditional financing institution would do and includes spending time at the companies, frequent meetings with managers, and being involved in the definition of the companies' strategies, hiring decisions (Hellmann and Puri, 2001), and top management compensation (Kaplan and Stromberg, 2000). In addition, venture capitalists bring their experience in evaluating the prospects of start-ups through their screening of potential investments (Hall and Hofer, 1993), their collaboration with other start-ups, their understanding of the solutions to the problems that these firms may face, and when stat-ups are best positioned to raise money (Gompers and Lerner, 1999). Finally they provide access to a strategic network including potential clients and suppliers, management talent (Bygrave and Timmons, 1992), strategic partners (Baum, Calabrese et.al; 2000). Our model tries to explore, at a first stage, the arbitrage between up-front funding and staged funding for the venture capitalist. Second, we analyze venture capitalists' choice between financing the project individually and splitting financing with other venture capitalists: syndicating venture investments.

THE MODEL

Consider an entrepreneur endowed with an innovative investment project. The project requires an initial investment, denoted I, and yields an outcome R. The entrepreneur is wealth-constrained. To implement the project, he must raise funds from outside investors. Some of those investors have special expertise in financing innovative projects. We call them *venture capitalists*. The innovative manager (denoted by M) relies on a venture capitalist (denoted by VC) for investment. We assume that the project requires a minimum amount I of capital investment. Thus, I is a given constant. The project lasts two periods. The venture capitalist provides a total investment I. The venture capitalist requires that the manager exerts an effort in exchange for financing the project. The manager's effort, e is applied during the two periods. The cost of effort for the manager is $c\ (e)$. The expected return of the project is function of the effort e of the manager during the two periods, and the monitoring intensity of the venture capitalist if monitoring occur. We suppose that the expected return is subject to a random effect z such that the realized return is $r = z.R$. At the beginning of the first period, the venture capitalist knows the distribution function of z, and the random effect is realized and publicly revealed at the beginning of the second period. The investment can be made in one amount, or in two stages I_0 and I_1. Although I is fixed, the venture capitalist can divide, in case of staged funding, the required amount into two parts,I_0 and I_1 for the two periods, and the venture capitalist also has the option to abandon the project ex post by not providing the remaining investment I_1.

In our model, staged financing dominates the alternative of giving the manager all the money at the same time, since staged financing enables the venture capitalist to cut his losses and to shut down underperforming projects early. Thus, the venture capitalist proposes to supply a total of I_0 in resources at the beginning of the first period. After the random effect is realized at the start of the second period, the venture capitalist thinks about supplying a total of I_1 in resources. If the project is neglected in the middle by either the manager or the venture capitalist, the project fails without any output and the initial investment is lost. If it is not the case, the project continues. To realize the return by the end of the second period, the necessary amount, I of investment must be made, i.e., $I = I_0 + I_1$. If both investments are made, the start-up continues to date *2* and the

entrepreneur and the venture capitalist can split the final return of the project. The split depends on their ownership percentages, which are assumed, fixed on the existing contract.

THE CONTRACT

In a typical agency model, contracts are typically based on some well-known aggregate measures. We assume that only the return of the project is contractible at the beginning of the first period (*ex ante*). We will concentrate on a certain type of informational asymmetry, having to do with the behavior of the manager during the investment. We shall assume that the agent's behavior is observable by the principal, but it is not verifiable. This asymmetric information problem is known as moral hazard. That effort is not verifiable means that it cannot be included in the terms of the contract. Then, the manager's effort is not verifiable. Concerning the venture capitalist's investment strategy, it's not contractible in the contract. The manager is steady with the reality that the VC normally has the option of abandoning the project in the future. Typically, venture capital investors provide capital unsecured by assets to young, private companies with the potential for rapid growth. Such investing covers most industries and is appropriate for businesses through the range of developmental stages. Investing in new or very early companies inherently carries a high degree of risk. But venture capital is long term or "patient capital" that allows companies the time to mature into profitable organizations. Venture capital financing consists in an equity financing, which is represented by funds that are raised by a business, in exchange for a share of ownership in the company. Equity financing allows a business to obtain funds without incurring debt, or without having to repay a specific amount of money at a particular time.

Taking into account the manager's effort e, and the random effect z, the venture capitalist proposes a sharing contract and finances the project strategically in two stages. As consequence, the contract proposed by the venture capitalist is a sharing contract $(\alpha, 1-\alpha)$ where α is the return share for the venture capitalist, and the remaining part $(1-\alpha)$ is the return share for the manager. At the end of the project, we will seek for the social optimum. The determination of a feasible social optimum amounts to maximize the collective utility function that is the individual utility combination.

We defined the optimal social welfare by:

$$SW = \prod_{M}^{*} + \prod_{VC}^{*}$$

Where:

SW : is the Social Welfare;

$\prod_{M}^{*}$: is the optimal profit level of the manager;

$\prod_{VC}^{*}$: is the optimal profit level of the Venture Capitalist.

The variables e, I_0, I_1 and α are determined endogenously in equilibrium.

Timing

The model has three periods: $t = 0, 1, 2$. All agents are risk neutral. There is no discounting. The timing of the actions is given below:

- At $t = 0$, the venture capitalist decides on an investment plan (I_0, I_1) (in case of up-front funding $I_1 = 0$ and $I_0 = I$). But first, the venture capitalist proposes a sharing contract $(\alpha,$

1 − α) to the manager; then if the manager accepts the sharing contract, the venture capitalist invests I_0, and the manager applies effort e and incurs cost *c (e)*. The venture capitalist may benefit from the resolution of uncertainty by investing ex post. However, ex post investment will impose a risk on the manager (from the venture capitalist's option to quit and to renegotiate), which may lead to a higher effort.

- At $t = 1$, the venture capitalist thinks about the decision to leave the project, to renegotiate the contract[1], or to provide the remaining investment. The uncertainty is resolved; then, if the project is good, the venture capitalist continues to invest in the project and offers the necessary investment I_1. If the project is average, the venture capitalist will ask for renegotiating a new contract. And if the project is bad, the venture capitalist will abandon the project without investing I_1.
- At $t = 2$, the project is finished, and the venture capitalist and the manager split the return based on the existing contract.

Assumptions

We suppose that venture capitalist will exert monitoring in case of up-front funding, and in case of staged financing, monitoring will not occur. In case of syndicated venture investments, venture capitalists will monitor the project.
Venture capitalist can ameliorate manager's moral hazard problem through monitoring. Venture capitalist chooses to monitor the project with an intensity m which has an impact on the manager's behavior. Monitoring is costly, an intensity m costs $C(m) = cm$ with $m \in [0, 1]$ and $c \in \left]0, \frac{1}{2}\right[$.

We suppose that the expected return is a linear function of the manager effort and the monitoring intensity. If the manager's effort is increasing, the expected return of the project is increasing, and if the monitoring intensity of the venture capitalist is high, the output expected of the project is high.
We assume that:

$$\frac{\partial R}{\partial e} \rangle 0 \text{ and } \frac{\partial R}{\partial m} \rangle 0$$

Thus, the realized return with z an observable random effect is:

$$r = \begin{cases} zR(e,m) \\ zR(e) \end{cases}$$

For simplicity, we consider the identity function for the expected return of the project.

[1] We will not consider the case of renegotiation in this paper.

We assume that the random effect is distributed according to the Uniform[2] distribution on interval $[0,1]$.

Moreover we assume that the effort cost function *c (e)* of the manager is such that:

$$c'(e) \rangle 0 \text{ and } c''(e) \geq 0$$

For example, we assume that:

$$c(e) = \frac{1}{\gamma} e^{\gamma} \text{ where } \gamma \geq 1$$

Venture Capital Funding

The basic need for funding in start-up firms comes from the entrepreneur's wealth constraints. The entrepreneur needs funds to finance the firm from beginning until it gives positive returns. Venture capitalists provide the funds required to overcome cash limitations during the initial stages of a firm's life, before the uncertainty of the venture is reduced and alternative sources of funding become available. Staged funding is a main characteristic of venture capital financing. Rather than providing their funding upfront, stage funding allows venture capitalists to periodically update their information about the firm, monitor its progress, review its prospects, and evaluate whether to provide additional funding or abandon the project. Staged financing provides venture capital with a real option. This option can be exercised or abandoned over time as the uncertainty about the start-up firm is reduced.

Staged financing is also advocated as a control mechanism. Theoretical models explain this financing structure as a governance mechanism to reduce the agency costs implicit in venture-backed start-ups. These theoretical models are important to understand the unique nature of venture financing. Neher (1999) studies the superiority of staged funding over up-front funding in a perfect certainty, full information setting (thus without a real option). A wealth-constrained entrepreneur establishes an agency relationship with an investor who provides the funds required to start the business. The investment that the entrepreneur makes is partly sunk, thus the relationship is open to the hold-up problem associated with the entrepreneur renegotiating the contract after the investment has been made. Staging the investment can reduce this hold-up problem if the value of the venture's assets without the entrepreneur increases over time as the entrepreneur's specific knowledge is embedded in the assets of the firm. The cash constraints associated with follow-up rounds, while never binding in equilibrium, provide the bargaining power that the investor needs to reduce the costs associated with the hold-up problem.

Kockensen and Ozerturk (2002) adopt an incomplete contracting structure to endogenously derive the optimality of staged venture capital funding. Their model assumes a wealth-constrained entrepreneur and an initial investor who provides a first round of funding, but does not commit to further funding. Before the second round funding decision, new information becomes available. This information is available to both the entrepreneur and the inside investor but not to potential outside investors. Consequently, the inside investor has an informational advantage that allows him to offer better terms if the project remains attractive as well as capture

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[2] A uniform distribution is one for which the probability of occurrence is the same for all values $z \in Z$.

a surplus associated with the private information. Staged funding provides the inside investor with a surplus that otherwise he would not capture. As in previous models, the first round removes the initial cash constraint and allows the entrepreneur to grow its venture. The second round gives the venture capitalist bargaining power to extract additional rents. The cash constraint implicit in the second round is the threat that strengthens the venture capitalist bargaining position even if rational expectations ensure that it is never carried out. In Canada, syndicated investments yielded significantly higher returns than those by a single venture capitalist. Amit found that "unconnected investments had average annual rates of return on the order of 15% to 20% whereas the syndicated investments had average returns of about 35% to 39%". Wang and Zhou (2004) derive the advantage of staged financing over up-front financing in a situation where there is information asymmetry between the venture capital and the entrepreneur. The entrepreneur provides effort, new information becomes available over time and only the final output is contractible. In the staged funding solution, the venture capitalist keeps the option of abandoning the project if the new information is not attractive. Staged funding provides two benefits. First, it reduces the cost associated with the risk of bad information becoming available. And second, it decreases the costs of the moral hazard problem that emerges from the agency relationship established when the first investment occurs. The initial funding provides the resources that the wealth constrained entrepreneur needs. In contrast, follow up funding relies on the threat that the entrepreneur runs out of resources to curve down moral hazard, even if in equilibrium this threat is never carried out. These models separate an initial stage where the cash-constrained entrepreneur receives funding and where an agency relationship is established from follow-up rounds. These follow-up rounds rely on the threat to the entrepreneur of hitting his cash constraint to reduce agency costs.

Up-front Funding

In this paragraph, we study the alternative where the venture capitalist provides the funds required by the manager in one amount *I*. At the beginning of the first period, the venture capitalist has the choice between two possibilities: financing or not the project. In case of upfront funding the venture capitalist loses the option to abondon the project. Since the effort is not verifiable, the sharing contract and active monitoring serves as means of control of moral hazard problem to the venture capitalist.

The venture capitalist should choose the contract $(\alpha^*, (1-\alpha^*))$ that maximizes his expected profit and taking into account the consequences of this contract on the manager's decisions.

Definition 1: Let the manager's expected profit from the project be:

$$\prod_M = \int_0^1 (1-\alpha) z R(e,m) h(z).dz - c(e) \qquad (1)$$

Where z is the random effect distributed according to the uniform density distribution $h(z) = 1$ with mean $\frac{1}{2}$ and variance $\frac{1}{12}$. We assume that the manager's reservation utility is equal to zero $(\underline{\prod} = 0)$.

The manager's will accept the contract if and only if it gives him a profit no smaller than $\underline{\prod}$ that represents the utility the manager can obtain by breaking his relationship with the venture capitalist.

The participation constraint (the Individual Rationality constraint) thus can be written:

$$\prod_M \geq 0 \Leftrightarrow$$

$$\int_0^1 (1-\alpha) zR(e,m)h(z).dz \geq c(e) \qquad \textit{(IR)}$$

When the venture capitalist offers him a contract (α∗, (1 − α∗)), the manager chooses his effort

by solving the following program:

$$\underset{e\in E}{Max}\left\{\int_0^1 (1-\alpha) zR(e,m)h(z).dz - c(e)\right\}$$

The FOC gives us the second constraint that the venture capitalist has to take into account: it is the Incentive Constraint. Thus the incentive compatibility constraint can be written:

$$\frac{\partial}{\partial e}\int_0^1 (1-\alpha) zR(e,m)h(z)dz = c'(e) \qquad \textit{(IC)}$$

Definition 2: Let the venture capitalist's expected profit from the project in case of upfront funding be:

$$\prod_{VC}^{U} = \int_0^1 \alpha.zR(e,m)h(z).dz - C(m)$$

Then, the venture capitalist will choose the contract which resolves the following program:

$$\underset{\alpha,m\in[0,1]}{Max}\left\{\int_0^1 \alpha.zR(e,m)h(z).dz - C(m)\right\}$$

Subject to : *(P.1)*

$$\begin{cases}(IC)\\(IR)\end{cases}$$

From the (IC) constraint we obtain:

$$\alpha = 1 - 2e^{\gamma-1} \qquad (2)$$

The FOCs conditions are:

$$\begin{cases}\dfrac{\partial \prod_{VC}^{U}}{\partial m} = 0\\ \dfrac{\partial \prod_{VC}^{U}}{\partial e} = 0\end{cases} \Leftrightarrow \begin{cases}\dfrac{1}{2} - e^{\gamma-1} - c = 0\\ \dfrac{1}{2} - \gamma e^{\gamma-1} - m(\gamma-1)e^{\gamma-2} = 0\end{cases}$$

The FOCs conditions of the program above imply that the venture capitalist will exert monitoring (which means $m\rangle 0$) if and only if the manager will exert an effort level smaller than $\bar{e} = \left(\dfrac{1}{2\gamma}\right)^{\frac{1}{\gamma-1}}$ since $m = \dfrac{1-2\gamma e^{\gamma-1}}{(\gamma-1)e^{\gamma-2}}$.

Another conclusion is that the optimal effort level which makes the profit level of the venture capitalist optimal is:

$$e^* = \left(\frac{1}{2} - c\right)^{\frac{1}{\gamma-1}} \qquad (3)$$

By substituting e^* in α we obtain:

$\alpha^* = 2c$

And:

$$m^* = \frac{2}{\gamma - 1}\left(\frac{1}{2} - c\right)^{\frac{1}{\gamma-1}}\left(\frac{2}{1-2c} - 2\gamma\right)$$

m^* is positive if and only if: $1 \langle \gamma \langle \frac{1}{1-2c}$

In case of upfront funding, the second best solution is:

$$e^* = \left(\frac{1}{2} - c\right)^{\frac{1}{\gamma-1}}$$

$$m^* = \frac{2}{\gamma - 1}\left(\frac{1}{2} - c\right)^{\frac{1}{\gamma-1}}\left(\frac{2}{1-2c} - 2\gamma\right)$$

$$\alpha^* = 2c$$

$$\left(\Pi_{VC}^{U}\right)^* = \frac{1}{2}\alpha^*\left(e^* + m^*\right) - cm^* = ce^* = c\left(\frac{1}{2} - c\right)^{\frac{1}{\gamma-1}}$$

$$\left(\Pi_{M}^{U}\right)^* = \left(\frac{1}{2} - c\right)^{\frac{1}{\gamma-1}}\left[\frac{\gamma - 1}{\gamma} + \frac{2}{\gamma - 1}\left(\frac{2}{1-2c} - 2\gamma\right)\right]$$

$$SW = \left(\Pi_{M}^{U}\right)^* + \left(\Pi_{VC}^{U}\right)^*$$

$$= \left(\frac{1}{2} - c\right)^{\frac{1}{\gamma-1}}\left[\frac{\gamma - 1}{\gamma} + \frac{2}{\gamma - 1}\left(\frac{2}{1-2c} - 2\gamma\right) + c\right]$$

Proof. See Appendix 1

Staged Funding in Venture Capital

In this section we consider stage funding in venture capital. We assume that the venture capitalist will not exert monitoring, and that the manager's effort is not verifiable.A prominent characteristic of venture capital investing is its staging structure through sequential financing rounds. At each round of financing, a venture capital firm supplies new financial resources to the start-up. These rounds of financing are discrete events staged over the life of the company as a private entity. Rounds of funding are critical in the relationship between venture capitalists and the start-ups that they invest in. They are not a simple transfer of financial resources; they also involve the redefinition of the governance structure of the firm and provide a signal about its

prospects. This new ownership structure affects the control structure of the company as well as the payoffs of a future liquidity event.

At the beginning of the first period, the venture capitalist invests I_0, the manager applies effort e. At the beginning of the second period, the uncertainty is resolved. Given the investment I_0, the manager's effort e, and the resolved uncertainty z, the venture capitalist decides whether to continue investing in the project or to abandon it. We assume that the venture capitalist will accept to provide the remaining amount to the manager if and only if the random effect value is such that:

$$\alpha\left(zR(e)\right) \geq I_1$$

This means that the venture capitalist will accept to continue financing the project if her expected return is at least equal to I1. Let assume that:

$$\bar{z} = \frac{I_1}{\alpha R(e)}$$

If $z \geq \bar{z}$, the venture capitalist will provide the refinancing; otherwise the venture capitalist will stop investing in the project. Thus:

$$\begin{cases} z \geq \bar{z} \Rightarrow \text{The project continues} \\ z \langle \bar{z} \Rightarrow \text{The project is abandonned} \end{cases}$$

Definition 3: Let the venture capitalist's expected profit from the project be:

$$\prod_{VC} = \int_{\bar{z}}^{1} \left(\alpha.z.R(e) - I_1\right) h(z).dz - I_0 \qquad (4)$$

Formally, the venture capitalist's program to resolve is given by:

$$\underset{\alpha, e \in E}{Max} \left\{ \prod_{VC} = \int_{\bar{z}}^{1} \left(\alpha.zR(e) - I_1\right) h(z).dz - I_0 \right\}$$

Subject to :

$$\begin{cases} \prod_M \geq 0 & \text{(IR)} \\ \dfrac{\partial}{\partial e} \displaystyle\int_{\bar{z}}^{1} \left(1-\alpha\right) zR(e)h(z).dz = c'(e) & \text{(IC)} \\ I_0 + I_1 = I & \text{(RC)} \\ I_0 \geq 0, I_1 \geq 0 \text{ and } \alpha \in [0,1] & \text{(PC)} \end{cases}$$

This problem establishes that the venture capitalist maximizes the profit that she obtains from investing in the project, under the restriction that the manager is willing to accept the sharing contract. This condition is known as *the participation condition* (I.R). Moreover, the venture capitalist maximizes its profit subject to its *individual participation condition* $\left(z \geq \bar{z}\right)$, *the manager's incentive compatibility constraint* (I.C) and *the resources constraint* (R.C). *Positivity constraint* (P.C) must be taken into account. The following solution characterizes the equilibrium in the situation of staged financing.

In the case of stage funding, the second best solution is:

$$e^* = \left(\frac{1}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$$

$$\alpha^* = 1 - \frac{1}{\gamma}$$

$$I_1^* = \left(\frac{\gamma^2-1}{\gamma}\right)\left(\frac{1}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}\sqrt{\frac{\gamma-1}{\gamma+1}}$$

$$\left(\prod_{VC}^{SB}\right)^* = (\gamma-1)\left(\frac{1}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} - I$$

$$\left(\prod_{M}^{SB}\right)^* = 0$$

$$SW = \left(\prod_{VC}^{S}\right)^* = (\gamma-1)\left(\frac{1}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} - I$$

Proof. See Appendix 2.

Now assume that the manager's effort is verifiable. We suppose that the venture capitalist demand an effort level in the contract. The problem to resolve is:

$$\underset{\alpha, e\in E}{Max}\left\{\prod_{VC} = \int_{\underline{z}}^{1}(\alpha.zR(e) - I_1)h(z).dz - I_0\right\}$$

Subject to : (P.3)

$$\begin{cases}\prod_M \geq 0 & \text{(IR)}\\ I_0 + I_1 = I & \text{(RC)}\end{cases}$$

In case of staged funding and a verifiable effort, the first best solution is:

$$\alpha^* = 1 - \frac{1}{\gamma}$$

$$e^* = \left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}$$

$$I_1^* = (\gamma - 1)\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{1-\gamma}}\sqrt{1 - \frac{2}{\gamma}}$$

$$\left(\overline{\Pi}_{VC}^{FB}\right)^* = \left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{\gamma-1}{2}\right) - I$$

$$\left(\overline{\Pi}_{M}^{FB}\right)^* = 0$$

$$SW = \left(\Pi_{VC}^{S}\right)^* = \left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{\gamma-1}{2}\right) - I$$

Proof. See Appendix 3.

We will use this solution to find the optimal strategy for the venture capitalist: up-front funding with active monitoring, staged funding with a not verifiable effort, and syndicated investments.

Syndicated Venture Investments

We now turn to the equilibrium with multiple venture financing: syndicated venture funding. Syndication is the process whereby a group of venture capitalists will each put in a proportion of the amount of money needed to finance a small business. Two main competing sights exist as to why venture capitalists syndicate investments. The traditional side developed from finance theory considers syndication as a means of risk sharing via portfolio diversification (Bygrave, 1987, 1988) and Smith & Kiholm Smith, 2000). In contrast the resource based side views syndication as a means to share resources such as information in the selection (before the investment is made) and management (after the investment is made) of investments. Related to the resource based view, syndication is considered as a mean to assure, increase or reciprocate transaction flow of the venture capitalist. Consider now that we have numerous firms and venture capitalists. As before firms have access to an investment project each and need external funds to finance them. Only venture financing is available. Venture capitalists can decide either to finance firms on their own individual venture financing or to share financing with other venture capitalist: multiple venture financing (syndicated investments).

Manager's behavioral choices are not observable: there is a moral hazard problem. Firms receive financing only if venture capitalists expect non-negative profits .i.e. if they expect a return at least equal to the income from an alternative investment. Suppose that we have n venture capitalists (henceforth indexed by $j = 1...n$), each venture capitalist finances k projects (henceforth indexed by $i = 1...k$). The difference with the up-front funding depends on how venture capitalists share financing with other $(n - 1)$ venture capitalists so that it invests $\frac{I}{n}$ unit in each of the k projects. We assume that all the projects have the same distribution function of uncertainty. We can have free-riding problem because monitoring is privately costly and not

observable, each venture capitalist has an incentive to reduce its own effort and benefits from the other venture capitalists' monitoring. Moreover, there is a duplication of efforts because venture capitalists don't coordinate in the choice of their monitoring intensities. The idea is that monitoring delivers a public good, and all venture capitalists financing a start-up benefit from the higher return of the project.

The expected return of the project *i* is:

$$R(e_i, M_i) = e_i + M_i = e_i + \sum_{j=1}^{n} m_{ij}$$

Where e_i is the effort level exerted by the manager of the project *i* and M_i is the total monitoring intensities that all n venture capitalists exert on the project *i*. Since venture capitalists *(VCs)* financing the same project will act as only one *VC*, they will propose α_i to the manager in case of success. Then they will split α_i equally between them. Then:

$$\alpha_{ij} = \frac{\alpha_i}{n}$$

The venture capitalist j will demand α_{ij} to the manager i in case of success of the project. Thus the proportion of the manager *i* from the project is $(1 - n\alpha_{ij})$. Venture capitalist *j*'s expected profit from a project *i* is given by:

$$\prod_{VC_j}^{i} = \left[\int_0^1 \alpha_{ij}.z.R(e_i, M_i).h(z).dz \right] - cm_{ij}$$

$$= \frac{1}{2}\alpha_{ij}\left(e_i + \sum_{j=1}^{n} m_{ij} \right) - cm_{ij}$$

Venture capitalist *j*'s expected profit from the *k* projects is given by:

$$\prod_{VC_j}^{k} = \sum_{i=1}^{k}\left(\left[\int_0^1 \alpha_{ij}.z.R(e_i, M_i).h(z).dz \right] - cm_{ij} \right)$$

$$= \sum_{i=1}^{k}\left[\frac{\alpha_i}{2n}\left(e_i + \sum_{j=1}^{n} m_{ij} \right) - cm_{ij} \right] \qquad (5)$$

Where the first term represents the expected return from the k projects venture capitalist *j* finances, and the second term is the total cost of monitoring k projects. Venture capitalist chooses m_{ij} and α_i to maximize (5).

Venture capitalists can finance more projects and reach a greater degree of diversification than with individual venture financing. Each venture capitalist can finance k projects instead of one.

The expected return of each project depends on the monitoring of all *k* venture capitalists. Venture capitalists will demand the same share of return in case of success of the project *i*.
The program to resolve is:

$$\underset{\alpha_i, m_{ij}}{Max}\left\{\Pi_{VC_j}^{k} = \sum_{i=1}^{k}\left[\frac{\alpha_i}{2n}\left(e_i + \sum_{j=1}^{n} m_{ij}\right) - cm_{ij}\right]\right\}$$

Subject to :

$$\begin{cases} \Pi_M^i = \frac{1}{2}(1-\alpha_i)\left(e_i + \sum_{j=1}^{n} m_{ij}\right) - \frac{1}{\gamma}(e_i)^{\gamma} \geq 0 \\ \frac{\partial \Pi_M^i}{\partial e_i} = 0 \Leftrightarrow \frac{1}{2}(1-\alpha_i) = (e_i)^{\gamma-1} \\ \Pi_{VC_j}^i = \frac{1}{2}\alpha_{ij}\left(e_i + \sum_{j=1}^{n} m_{ij}\right) - cm_{ij} \geq 0 \end{cases}$$

From $\frac{\partial \Pi_M^i}{\partial e_i} = 0$, we obtain $\alpha_i = 1 - 2(e_i)^{\gamma-1}$. Taking into account this result into the objective function, we find:

$$\underset{\alpha_j, m_j}{Max}\left\{\Pi_{VC_j}^{k} = \sum_{i=1}^{k}\left[\frac{1}{n}\left(\frac{1}{2} - (e_i)^{\gamma-1}\right)\left(e_i + \sum_{j=1}^{n} m_{ij}\right) - cm_{ij}\right]\right\}$$

The third constraint means that the venture capitalist *j* will monitor the project *i* if its expected profit is positive, which gives us:

$$m_{ij} \leq \frac{1}{2c}\alpha_{ij}\left(e_i + \sum_{j=1}^{n} m_{ij}\right)$$

The total monitoring intensities that VCs exert for the project *i* they finance is:

$$M_i = \sum_{j=1}^{n} m_{ij}$$

The FOCs Conditions give $e_i = \left(\frac{1}{2} - c\right)^{\frac{1}{\gamma-1}}$. The average monitoring intensity exerted by the VCs on the project *i* is $M_i = \frac{1 - 2\gamma e_i^{\gamma-1}}{2n(\gamma-1)e_i^{\gamma-2}}$. Venture capitalists will exert monitoring if the manager's effort is lower than $\overline{e_i} = \left(\frac{1}{2\gamma}\right)^{\frac{1}{\gamma-1}}$, which maximizes their expected profit. We notice that the necessary total monitoring intensity to control manager's misbehavior is lower in case of syndication than in case of single venture funding (here up-front funding) since monitoring is dispersed through n venture capitalists providing finance to the manager. The following proposition characterizes the equilibrium of the syndicated investment. The unique symmetric equilibrium of the syndicated venture investment, in which each venture capital monitors the project with intensity m_{ij} and demands the share of return α_{ij} is given by:

$$\alpha_{ij}^{*} = \frac{2c}{n}$$

$$e_i^{*} = \left(\frac{1}{2} - c\right)^{\frac{1}{\gamma-1}}$$

$$M_i^{*} = \left(\frac{1}{2} - c\right)^{\frac{1}{\gamma-1}}\left(\frac{\gamma c}{n(\gamma-1)} - \frac{1}{2n}\right)$$

$$\overline{m_{ij}} = \left(\frac{1}{2} - c\right)^{\frac{1}{\gamma-1}}\left(\frac{\gamma c}{n(\gamma-1)} + \frac{1}{2n}\right)$$

$$\left(\prod_{VC}^{SyF}\right)^{*} = \begin{cases} 0 \text{ if } m_{ij}^{*} = \overline{m}_{ij} \\ \rangle 0 \text{ if } m_{ij}^{*} \langle \overline{m}_{ij} \end{cases}$$

$$\left(\prod_{M}^{SyF}\right)^{*} = \left[\left(\frac{1}{2} - \frac{c}{n}\right)\left(\frac{2n-1}{2n} + \frac{\gamma c}{n(\gamma-1)}\right) - \left(\frac{1}{2\gamma} - \frac{c}{\gamma}\right)\right]\left(\frac{1}{2} - \frac{c}{k}\right)^{\frac{1}{\gamma-1}}$$

$$SW = \left(\prod_{VC}^{SyF}\right)^{*} + \left(\prod_{M}^{SyF}\right)^{*}$$

$$= \left[\left(\frac{1}{2} - \frac{c}{n}\right)\left(\frac{2n-1}{2n} + \frac{\gamma c}{n(\gamma-1)}\right) - \left(\frac{1}{2\gamma} - \frac{c}{\gamma}\right)\right]\left(\frac{1}{2} - \frac{c}{k}\right)^{\frac{1}{\gamma-1}}$$

If the Venture capitalist will choose to monitor the project closely and with a high monitoring intensity such that $m_{ij} = \left(\frac{1}{2} - c\right)^{\frac{1}{\gamma-1}}\left(\frac{\gamma c}{n(\gamma-1)} + \frac{1}{2n}\right)$ her expected profit will be $\prod_{VC}^{SyF} = \left(\frac{1}{2} - \frac{c}{k}\right)^{\frac{1}{\gamma-1}}\left(\frac{\gamma c}{n(\gamma-1)}\right)\left(\frac{1}{n} - 1\right) \leq 0$. The VC's objective is to maximize its expected profit, she will choose to provide a lower monitoring level since the lead venture capitalist will monitor the project closely (Free-riding problem), thus:

$$m_{ij}^{*} \langle \left(\frac{1}{2} - c\right)^{\frac{1}{\gamma-1}}\left(\frac{\gamma c}{n(\gamma-1)} + \frac{1}{2n}\right)$$

Since VCs don't operate cooperatively, all VCs will provide a monitoring intensity such that: $m_{ij}^{*} = \overline{m}_{ij}$, and the only agent who will profit from the free-riding problem is the manager.

Proposition 1: *Even when the number of venture capitalists financing many projects simultaneously is high, the net profit of the manager is positive.*

$$\lim_{n \to +\infty, k \to +\infty} \left(\prod_{M}^{SyF}\right)^{*} = \left(1 + \frac{c}{\gamma} - \frac{1}{2\gamma}\right)\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}$$

In syndicated venture investments with active monitoring, the venture capitalist does not obtain positive returns. This is due to the problem of free-riding. The net profit of the manager in case of syndicated investments with active monitoring is lower than in case of up-front funding.

Analysis

To appreciate the role of venture capitalist financing strategy in alleviating moral hazard, we consider the following cases: up-front funding, staged funding and syndicated investments. We will use UF, SF and Syn for up-front funding, staged funding and syndication, respectively.
If we compare the social welfare when the cost of monitoring goes to 0 and when the cost of monitoring is high, and goes to $\frac{1}{2}$. We find:

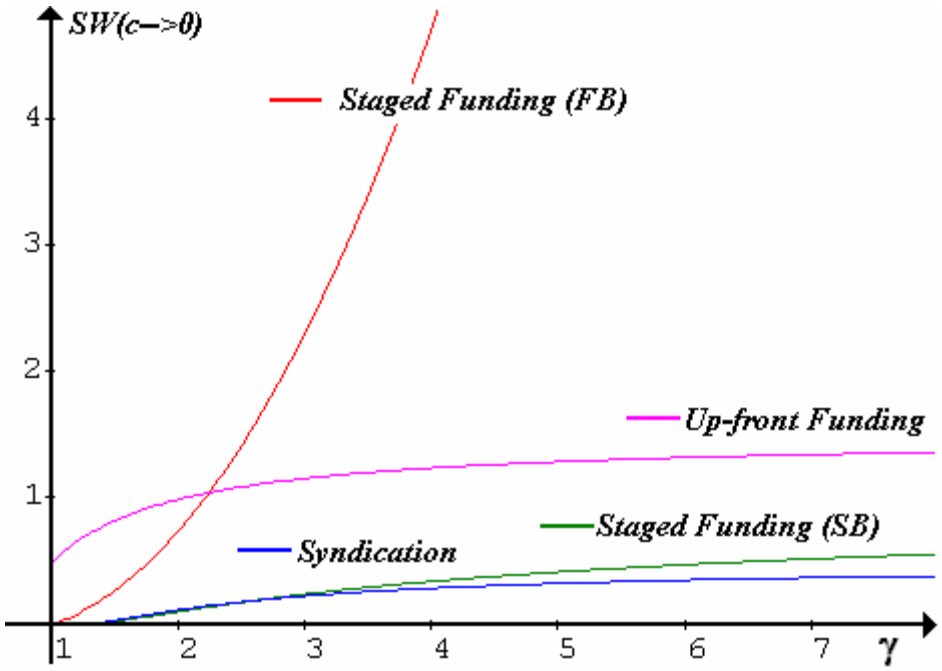
	SW (c→0)	*SW (c→$\frac{1}{2}$)*
Up-front funding	$\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\left[1-\frac{1}{\gamma}\right]+\frac{1}{2}$	$\frac{1}{2}$
Staged funding (FB)	$\left(\frac{\gamma^2-1}{2\gamma}\right)\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}$	$\left(\frac{\gamma^2-1}{2\gamma}\right)\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}$
Staged funding (SB)	$(\gamma-1)\left(\frac{1}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$	$(\gamma-1)\left(\frac{1}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$
Syndication	$\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\left[1-\frac{1}{2n}-\frac{1}{\gamma}\right]$	$\left[\frac{n-1}{2n}+\frac{n-1}{4n^2(\gamma-1)}\right]\left(\frac{k-1}{2k}\right)^{\frac{1}{\gamma-1}}$

Proposition 2: *When γ goes to infinity, the Social Welfare in case of up-front funding is higher then in case of syndication, when the cost of monitoring is high, and when the cost of monitoring is low, syndicated investments increase the social welfare of the agents.*

The proposition above suggests that in case of syndicated venture investments the social profit of the agents is higher than in case of a single venture funding (up-front funding) when monitoring is not costly. VCs will monitor closely the project. Otherwise, syndication is not the optimal strategy for the venture capital investor when monitoring is costly, since monitoring is not an observable action between the VCs; they will free-ride to maximize their expected profit. Thus, when monitoring is costly, up-front funding with active monitoring is an optimal strategy. Monitoring intensities are higher when there is only one venture capitalist that finances the project, this is the direct result of the numerous venture capitalist that provide funding to the start-up. As in up-front funding, the manager of the project i must provide an effort level lower than $\bar{e}_i = \left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}$ to give incentives for the VCs to monitor the project.

The comparison of four social welfare curves when the cost of monitoring is low is shown in Figure 1.

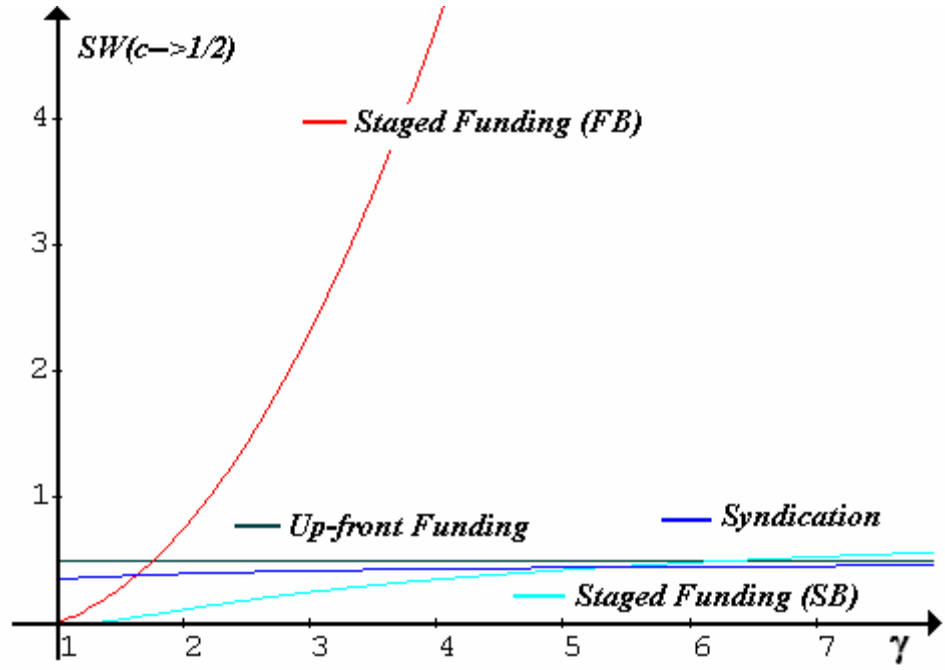
Figure 1



The figure above indicates that the social welfare in case of up-front funding with active monitoring when γ is low, which means that the cost of the effort provided by the manager is higher than the social welfare for the first best solution. The syndicated investments provide the lower social welfare; this is the direct result of free-riding problem. We conclude that when providing effort is costly for the manager, up-front funding with active monitoring is the optimal strategy to obtain higher results. In the other case, when supplying effort isn't costly ($\gamma \to \infty$), staged funding with verifiable effort is the optimal strategy.

The comparison of four social welfare curves when the cost of monitoring is high is shown in Figure 2.

Figure 2



The social welfare in case of up-front funding with active monitoring is higher than the social welfare in case of staged funding for given value of γ (the cost of effort converges to zero as $\gamma \rightarrow +\infty$).

In fact active monitoring will have an impact on the manager's behavior which gives an increasing in the net profit of the agents. Staged funding can reduce uncertainty and alleviate moral hazard. In our model, the monitoring in up-front funding increase the net profit of the venture capitalist and the start-up. For lower value of γ, which means a high cost of effort for the manager, the VC must monitor the project closely to control the manager's misbehavior. In other words, the manager will not provide a higher effort since it is costly. Monitoring will be a good mechanism to control this problem. But when γ is high, the cost of the effort is low, and the manager will provide a higher effort to increase the expected return of the project and then there is no reason for the VC to monitor closely the project. For that reason, the social welfare in case of staged funding (FB) is higher than the social welfare in case of up-front funding with active monitoring. When the cost of monitoring is high, the social welfare in case of syndicated investments approaches the social welfare in case of up-front funding: venture capitalists will operate as one "financier".

CONCLUSION

This study has extended the venture capital literature by analyzing the role of asymmetric information conditions, which may arise subsequent to the time of contracting, can play in the choice of contract type. However, the entrepreneur has an incentive to behave opportunistically under asymmetric information. In our model we consider an entrepreneur facing an imperfect capital market and a venture capital investor facing uncertainty and moral hazard. We study how

active monitoring can increase the expected net profit of these two agents, and thus the social welfare. Under certain assumptions, we obtain some unique results on the performance and role of the monitoring that the venture capitalist uses to mitigate moral hazard problem and to reduce uncertainty. We find that up-front funding may be socially better choice when the cost of the effort is high. In particular monitoring in up-front funding can eliminate the incentives of the manager to misbehave since providing effort is costly. Moreover, this paper analyzes VC's incentives to enter in syndicated investments with other VCs in a context where both start-ups and VCs are subject to moral hazard, and monitoring is essential. VCs choose syndicated venture investments whenever the benefit of a greater diversification in terms of higher overall monitoring dominates the drawbacks of free-riding problem. We find that when the number of VCs financing the same project is high the social welfare in case of syndication is lower than in case of up-front funding.

We develop the analysis under the assumption that all VCs share financing equally when they syndicate the investment and receive the same share of benefits in case of success of the project. Allowing for asymmetric shares would lead to different results. How the VCs split the investment amount between them? Using a bargaining model, we can have an answer to this idea. This analysis constitutes an interesting avenue for future research.

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Appendix

Proof. Solution 1

For $z \to U[0,1]$, we have $h(z) = 1$

Problem (P.1) to resolve is:

$$\underset{\alpha,m}{Max}\left\{\prod_{VC}^{U} = \frac{1}{2}\alpha(e+m) - cm\right\}$$

Subject to :

$$\begin{cases} \frac{1}{2}(1-\alpha) = e^{\gamma-1} \\ \frac{1}{2}(1-\alpha)(e+m) \geq \frac{1}{\gamma}e^{\gamma} \end{cases}$$

The *(IC)* constraint implies the *(IR)* condition; we can consider the following program:

$$\underset{\alpha,m}{Max}\left\{\prod_{VC}^{U} = \frac{1}{2}\alpha(e+m) - cm\right\}$$

Subject to :

$$\frac{1}{2}(1-\alpha) = e^{\gamma-1}$$

Which is equivalent to:

$$\underset{\alpha,m}{Max}\left\{\prod_{VC}^{U} = \frac{1}{2}(e+m) - me^{\gamma-1} - e^{\gamma} - cm\right\}$$

The FOCs conditions are :

$$\begin{cases} \frac{1}{2} - e^{\gamma-1} - c = 0 \\ \frac{1}{2} - \gamma e^{\gamma-1} - m(\gamma-1)e^{\gamma-2} = 0 \end{cases}$$

Which gives e^{*}, m^{*}, then α^{*} and $\left(\prod_{VC}^{U}\right)^{*}$ in solution 1.

Proof. Solution 2

We know that: $\bar{z} = \dfrac{I_1}{\alpha R(e)}$

$$\prod_M = \int_{\bar{z}}^{1} (1-\alpha) z R(e) h(z) dz - c(e)$$

$$= \frac{1}{2}(1-\alpha)e - \frac{(1-\alpha)I_1^2}{2\alpha^2 e}$$

$$\frac{\partial \prod_M}{\partial e} = \frac{1}{2}(1-\alpha) + \frac{(1-\alpha)I_1^2}{2\alpha^2 e^2}$$

And:

$$\prod_{VC} = \int_{\bar{z}}^{1} \alpha z R(e) h(z) dz - I_0$$

$$= \frac{1}{2}\alpha e - \frac{I_1^2}{2\alpha e} - I$$

The problem (P.2) to resolve is:

$$\underset{\alpha, e \in E}{Max} \left\{ \prod_{VC} = \frac{1}{2}\alpha.e - \frac{I_1^2}{2\alpha.e} - I \right\}$$

Subject to :

$$\begin{cases} \dfrac{1}{2}(1-\alpha) + \dfrac{(1-\alpha)I_1^2}{2\alpha^2 e^2} = e^{\gamma - 1} \\ \dfrac{1}{2}(1-\alpha)e - \dfrac{(1-\alpha)I_1^2}{2\alpha^2 e} \geq \dfrac{1}{\gamma} e^{\gamma} \end{cases}$$

The two first constraints give us:

$$\alpha = 1 - \left(\frac{1+\gamma}{\gamma} \right) e^{\gamma - 1}$$

And:

$$I_1 = \left(e - \frac{\gamma + 1}{\gamma} e^{\gamma} \right) \sqrt{\frac{\gamma - 1}{\gamma + 1}}$$

Thus,

$$\underset{\alpha, e \in E}{Max} \left\{ \prod_{VC} = \frac{\gamma}{\gamma + 1} e - e^{\gamma} - I \right\}$$

The FOC for *e* implies:

$$\frac{\partial \prod_{VC}}{\partial e} = 0 \Leftrightarrow e^* = \left(\frac{1}{\gamma + 1} \right)^{\frac{1}{\gamma - 1}}$$

Which immediately implies α^*, I_1^*, and $\left(\prod_{VC}^{SB} \right)^*$ in solution 2.

Proof. Solution 3

The problem *(P.3)* to resolve is:

$$\underset{\alpha,e\in E}{Max}\left\{\prod_{VC}^{U}=\frac{1}{2}\alpha.e-\frac{I_1^2}{2\alpha.e}-I\right\}$$

Subject to :

$$\begin{cases}\frac{1}{2}(1-\alpha)e-\frac{(1-\alpha)I_1^2}{2\alpha^2 e}\geq\frac{1}{\gamma}e^{\gamma}\\ I=I_0+I_1\end{cases}$$

Considering that:

$$\frac{1}{2}(1-\alpha)e-\frac{(1-\alpha)I_1^2}{2\alpha^2 e}=\frac{1}{\gamma}e^{\gamma}$$

We obtain:

$$\frac{I_1^2}{\alpha^2}=e^2-\frac{2}{\gamma(1-\alpha)}e^{\gamma+1}$$

And:

$$I_1=\alpha\sqrt{e^2-\frac{2}{\gamma(1-\alpha)}e^{\gamma+1}}$$

The program to resolve is:

$$\underset{\alpha,e\in E}{Max}\left\{\prod_{VC}=\alpha.e-\frac{\alpha}{\gamma(1-\alpha)}e^{\gamma}-I\right\}$$

FOCs are :

$$\begin{cases}\frac{\partial\prod_{VC}}{\partial\alpha}=0\\ \frac{\partial\prod_{VC}}{\partial e}=0\end{cases}\Leftrightarrow\begin{cases}\alpha-\frac{\alpha}{1-\alpha}e^{\gamma-1}=0\\ e-\frac{1}{\gamma(1-\alpha)^2}=0\end{cases}$$

Which gives:

$$\alpha^*=1-\frac{1}{\gamma}$$

Which immediately implies e^*, I_1^*, and $\left(\prod_{VC}^{FB}\right)^*$ in solution 3.

Proof. Solution 4

The problem *(P.4)* to resolve is:

$$\underset{\alpha_i, m_{ij}}{Max}\left\{\prod_{VC_j}^{k} = \sum_{i=1}^{k}\left[\frac{\alpha_i}{2n}\left(e_i + \sum_{j=1}^{n} m_{ij}\right) - cm_{ij}\right]\right\}$$

Subject to :

$$\begin{cases} \prod_M^i = \frac{1}{2}(1-\alpha_i)\left(e_i + \sum_{j=1}^{n} m_{ij}\right) - \frac{1}{\gamma}(e_i)^{\gamma} \geq 0 \\ \frac{\partial \prod_M^i}{\partial e_i} = 0 \Leftrightarrow \frac{1}{2}(1-\alpha_i) = (e_i)^{\gamma-1} \\ \prod_{VC_j}^i = \frac{1}{2}\alpha_{ij}\left(e_i + \sum_{j=1}^{n} m_{ij}\right) - cm_{ij} \geq 0 \end{cases}$$

The second constraint provides us:

$$\alpha_i = 1 - 2(e_i)^{\gamma-1}$$

Considering that VC will choose the monitoring intensity which gives him a profit at least equal to zero. Then:

$$m_{ij} = \frac{1}{2c}\alpha_{ij}\left(e_i + \sum_{j=1}^{n} m_{ij}\right)$$

The FOCs conditions are:

$$\begin{cases} \frac{\partial \prod_{VC}^{SyF}}{\partial m_{ij}} = 0 \\ \frac{\partial \prod_{VC}^{SyF}}{\partial \alpha_{ij}} = 0 \end{cases} \Leftrightarrow \begin{cases} \frac{n}{2}\sum_{i=1}^{k}\alpha_{ij} - c = 0 \\ \left(\frac{1}{2} - \frac{1}{2}n\alpha_{ij}\right)^{\frac{1}{\gamma-1}}\left(1 - \frac{\alpha_{ij}}{\gamma-1}\left[\frac{1}{2} - \frac{1}{2}n\alpha_{ij}\right]^{-1}\right) + nm_{ij} = 0 \end{cases}$$

From the FOCs for m_{ij} and e_i we obtain:

$$\begin{cases} \frac{\partial \prod_{VC}^{SyF}}{\partial m_{ij}} = 0 \\ \frac{\partial \prod_{VC}^{SyF}}{\partial \alpha_{ij}} = 0 \end{cases} \Leftrightarrow \begin{cases} \frac{1}{2} - (e_i)^{\gamma-1} - c = 0 \\ \frac{1}{2n} - \frac{1}{n}\gamma(e_i)^{\gamma-1} - \overline{M}_i(\gamma-1)(e_i)^{\gamma-2} = 0 \end{cases}$$

This gives:

$$(e_i)^* = \left(\frac{1}{2} - c\right)^{\frac{1}{\gamma-1}} \text{ and } M_i = \frac{1 - 2\gamma(e_i)^{\gamma-1}}{2n(\gamma-1)(e_i)^{\gamma-2}}$$

This immediately implies $(\alpha_i)^*$ and $(\alpha_{ij})^*$.